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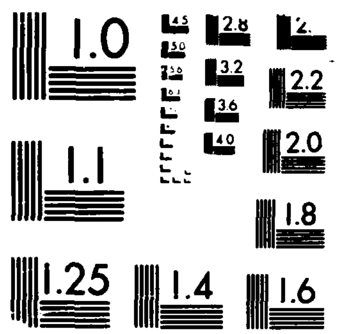

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

closed arithmetic; computer arithmetic; difference equations; differential equations; elementary functions; error analysis; floating-point system; hardware; level-index arithmetic; Newton's rule; overflow; polynomial evaluation; precompiler; programming; recurrence relations; software; surface-fitting; symmetric level-index system; transcendental equations; transcendental mathematical (cont'd)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Problems were studied and solved in three main areas: unrestricted algorithms for mathematical functions; error analysis of numerical algorithms; new computer arithmetics.

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19. Key Words (continued)

functions; underflow; unrestricted algorithms.

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FINAL REPORT

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RESEARCH FINDINGS

I. Unrestricted computing algorithms for the elementary functions and special functions of mathematical physics.

Publications*:[1], [4].

"Unrestricted algorithms for mathematical functions" is a term that was coined in the course of work supported by a previous AROD Contract (DAAG 29-80-C-0032). It refers to any algorithm that will generate function values to any guaranteed precision for any values of the variables. The object is two-fold. Firstly, constructive numerical algorithms are sought that could be used, for example, to produce restricted algorithms based on rational approximations or in expansions series in Chebyshev polynomials. Secondly, new mathematical and computational tools are to be developed for constructing the very demanding unrestricted algorithms.

During the present contract unrestricted algorithms were devised and published for reciprocals and square roots. Progress was also made on the trigonometric and logarithmic functions analogous to those published in [14] for the exponential function. However, it has become increasingly clear during the course of this work that the floating-point number system has serious drawbacks for constructing unrestricted algorithms, including, for example, an inappropriate error measure. Much more satisfactory number systems are now under development, especially the level-index and symmetric level-index systems described in III below. In consequence, further work on

* References are listed on pp. 6-7.

unrestricted algorithms has been suspended until a precompiler for the new number systems becomes available.

II. Error analysis of numerical algorithms.

Publications: [2], [5], [10], [11].

All of the objectives described in the research proposal (20606-MA) for the contract were achieved with the exception of the development of high quality software for solving systems of linear algebraic equations complete with strict error bounds. Partly this was because of lack of suitable assistance on the programming side, and partly because of the development of the IBM ACRITH package which covers similar ground; see [18], [21].

In the case of linear recurrence relations of the form

$$a_r p_{r+1} = b_r p_r + c_r p_{r-1}$$

methods for constructing strict error bounds of both a posteriori and a priori types in $O(r)$ arithmetic operations were developed and tested. The results should prove to be useful in the construction of robust software for the generation of transcendental mathematical functions.

New error bounds of a posteriori type were constructed for the evaluation of polynomials and their derivatives, and for the solution of transcendental equations by Newton's rule, both for real and complex variables.

III. New computer arithmetics.

Publications: [1], [3], [6], [7], [8], [9], [11], [12]. See also [15], [16], [20].

The floating-point system emerged as the standard form of computer arithmetic once it became clear that the effort of scaling computations to stay within representable ranges of the fixed-point system would place an intolerable burden on programmers. The floating-point system has served the computing community well; it is difficult to imagine a more satisfactory number system with the technology that has been available hitherto. However, the system has a major disadvantage: it is not closed under the four basic arithmetic operations of addition, subtraction, multiplication and division (excluding division by zero). In consequence, there will always be computations that cannot be executed because of overflow or underflow failure.

Is it possible to construct a natural closed system of arithmetic? If so, could it be executed in a reasonably economic manner with today's computer technology? Would there be other advantages or disadvantages when compared with the floating-point system? Each of these questions has been answered, at least in part, by recent work of the proposer and his colleagues in the U.S. and U.K. Two new systems of arithmetic, called the level-index and symmetric level-index systems have been constructed and analyzed. These systems are based on the representation of numbers by their generalized logarithms. The systems are closed under arithmetic operations in finite-precision arithmetic (other than division by zero, of course). They have other advantages, including a redistribution of local precision and a natural error measure that includes absolute precision and relative precision as special cases.

So far, the level-index and symmetric level-index systems have been implemented in software. Moreover, much progress has been made in adapting the precompiler devised by F. Crary [17]. A big advantage of Crary's precompiler is that it was mostly written in Fortran. The Fortran was the old standard version, and where necessary we have rewritten it so as to comply fully with the current Fortran standard [13]. Eventually the resulting software system will permit a user to write a program that is similar in style to Fortran and capable of execution on any computer with a standard Fortran compiler. The system will provide the capability of specifying floating-point, level-index, symmetric level-index or other arithmetics together with a choice of working precisions.

The apparent disadvantage of the new number systems is the complicated nature of their arithmetic operations, leading to slow execution times. However, even this disadvantage is beginning to be overcome. By means of a surface-fitting approach it may well prove possible to achieve execution speeds, in hardware, within a factor of 2 or 3 of the corresponding floating-point arithmetic operations. In this event, in addition to freedom from failure caused by overflow or underflow, many computations would actually run faster in level-index or symmetric level-index arithmetic because of the ability to perform them with simpler algorithms and programs. Furthermore, vast amounts of human time and effort in writing and debugging programs and software would be saved.

In brief, there is the potential now for an impact on computing and numerical analysis in the 21st Century that would be comparable to that caused by the switch from fixed-point to floating-point hardware in the 1950's. Not least affected, because of the greater freedom from possible failure, would be software used for military purposes by the Army and other agencies of the Department of Defense.

IV. Uniform asymptotic solutions of linear differential equations.

This was the fourth objective described in the original proposal, but in fact it was completed successfully under an extension of the previous contract. The work is described in the final report on Grant DAAG 29-77-G-0003 and Contract DAAG 29-80-C-0032. The only subsequent development was the award, in June 1984, of the Ph.D. degree in Applied Mathematics to J. J. Nestor, Jr., for his thesis research on the problem of a coalescing simple turning point and simple pole [19].

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